PROPERTIES OF TRIANGLES

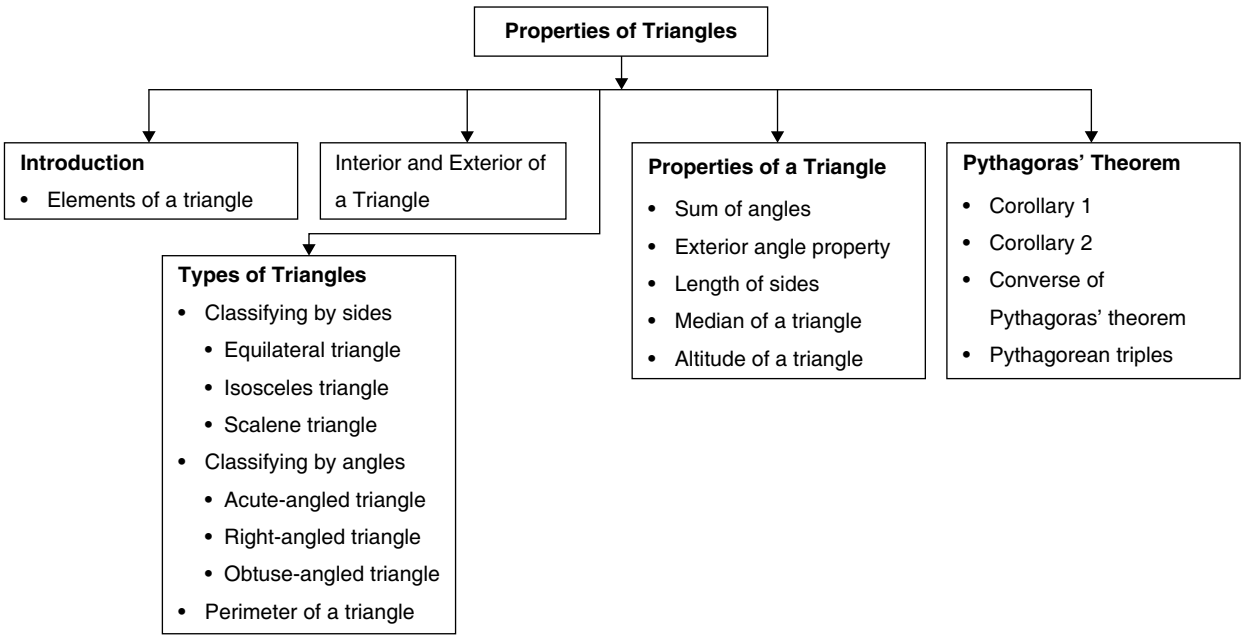
Learning objectives

Students will learn:

- To define a triangle and recognise its elements and properties
- To classify triangles based on angles and based on sides
- To state theorems related to sum of angles, exterior angle and lengths of sides of triangles
- To state and prove Pythagoras' theorem
- To form Pythagorean triples
- To apply Pythagoras' theorem to other regular polygons

Prior knowledge

- Closed figures formed by line segments
- Angles and types of angles
- Interior and exterior region of a closed figure
- Linear pair/Supplementary angles



Guidelines to teach

Rewind

- Ask students to complete the Rewind exercise.

Warm up

- Ask students to find the sum of the angles in a triangle as given in the course book.

Introduction

- Elicit from students what the prefix *tri* means in words like tripod, tricycle, tricolour, trio, trident and so on.

- Elicit from them that a triangle is a figure with three angles and three sides.

Elements of a triangle

- Draw a triangle on the board or direct the attention of students to the triangle illustrated for the topic in the course book. Instruct students to identify and name its three sides and three angles.
- Elicit from students the three vertices of the triangle. Ask them to identify the side opposite each vertex.
- Ask students:
- If a side of ΔABC is denoted by *a,* which side does it refer to? Which vertex of the triangle is it opposite to? (*BC, A*)
- If a side of ΔABC is denoted by *b,* which side does it refer to? Which vertex of the triangle is it opposite to? (*CA, B*)
- If a side of ΔABC is denoted by *c,* which side does it refer to? Which vertex of the triangle is it opposite to? (*AB,* C)

Interior and exterior of a triangle

- Explain the terms interior and exterior of a triangle with the help of the course book.
- Ask students to complete the statements:
  - A triangle divides the plane on which it is drawn, into _____ parts. *(three)*
  - Points that lie inside the triangle are in the _____ of the triangle. *(interior)*
  - Points that lie outside the triangle are in the part of the plane called the _____ of the triangle. *(exterior)*
  - Points that lie _____ form the boundary of the triangle or the triangle itself. (*on the triangle*)
  - The _____ and the _____ of the triangle form the triangular region. *(boundary, interior)*

Types of triangles

- Elicit from students the elements of a triangle.
- Tell them that triangles can be classified into different types based on their angles and sides.

Classifying by sides

Equilateral triangle

- Explain the properties of an equilateral triangle using the course book.
- Direct the attention of students to the equilateral triangle ABC in the course book.
- Ask them to measure each of its angles.
- Help them conclude that each angle of an equilateral triangle is equal to 60°.

***Isosceles triangle*:**

- Explain the properties of an isosceles triangle using the course book.
- Ask students to measure the angles of each of the three isosceles triangles given in the course book.
- Help them conclude that the angles opposite to the equal sides are also equal.

***Scalene triangle*:**

- Use the course book to explain the properties of a scalene triangle.
- Ask students to draw a triangle of three unequal sides, say 5 cm, 6 cm and 7 cm and measure
  their angles.
- Help them conclude that no two angles of a scalene triangle are equal.
- Ask students to complete the statements:
  - Angles opposite to equal sides in a triangle will be _____. *(equal)*
  - In an _____ triangle, all the three angles are equal. *(equilateral)*
  - In an _____ triangle, two angles are equal. *(isosceles)*

Classifying by angles

***Acute-angled triangle*:**

- Explain the properties of an acute–angled triangle using the course book.

***Right-angled triangle*:**

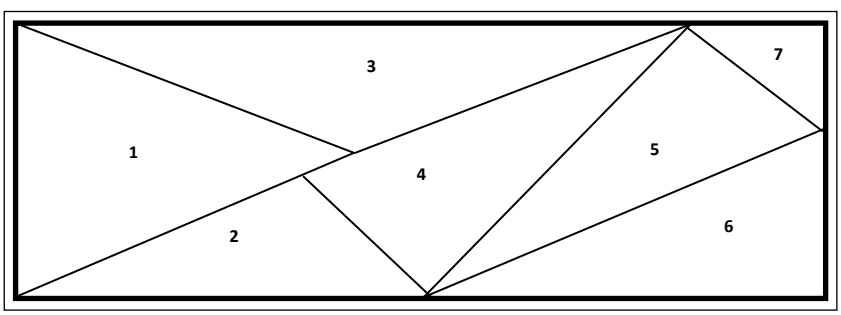
- Use the course book to explain the properties of a right-angled triangle.

***Obtuse-angled triangle*:**

- Explain the properties of an obtuse-angled triangle using the course book.
- Instruct students to complete the Quick Check exercise.

1. Extension activity for types of triangles

- Divide the class into convenient groups.
- Make as many copies of the figure and the combinations (a) to (g) as the number of groups.



(a) Isosceles and acute
(b) Scalene and right
(c) Equilateral and acute
(d) Isosceles and right
(e) Scalene and acute
(f) Isosceles and obtuse
(g) Scalene and obtuse

- Give each group a copy.
- Ask students to choose the correct description of each triangle numbered 1–7 from the combinations (a) – (g). Ask them to write the description against the correct numbers.
- Check the work of all groups and point out corrections, if any.

Perimeter of a triangle

- Elicit the definition of perimeter of a triangle from students.
- Ask them to attempt the Tryout given in the course book for the elements of a triangle, interior and exterior of a triangle and the types of triangles.

Properties of a triangle

Sum of angles

- Instruct students to do the Activity to verify that the sum of the angles of a triangle is always equal to 180°.
- Explain the proof for this property as given in the course book.
- You may ask students these questions:
  - Which type of triangle—equilateral, scalene, isosceles—can have three angles of the same measure? (*equilateral*)
  - What combination of angles can we have in an obtuse-angled triangle? Choose from:
    - Obtuse, acute, acute • Obtuse, right, acute • Obtuse, obtuse, acute • Obtuse, right, right
  - In an isosceles right-angled triangle, what is the measure of the two angles other than the right angle? *(45°)*

Exterior angle property

- Use the illustration in the course book to explain the terms exterior angle. Explain the terms interior adjacent angle and interior opposite angles (or remote interior angles) of an exterior angle.
- State the exterior angle property of a triangle and explain the proof of this property as given in the course book.
- Ask students to complete the statements:
  - An exterior angle of a triangle is formed by producing any of its _____. *(sides)*
  - The measure of an exterior angle of a triangle is equal to the sum of its _____. (*interior opposite angles*)
  - The exterior angle of a triangle formed by producing any of the sides of a triangle is _____ than either of its interior opposite angles. (*greater*)
  - An exterior angle of a triangle and _____ form a linear pair. (*its interior adjacent angle*)

- The sum of an exterior angle of a triangle and its interior adjacent angle is _____. (*180°*)

Length of its sides

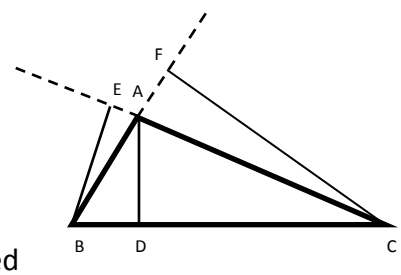
- Help students verify that a triangle can be drawn only if the sum of any two sides is greater than the third side from the Activity.
- Ask them to complete the statement:
  - The sum of any two sides of a triangle is always _____ than the third side. (*greater*)

Median of a triangle

- Use the course book to explain the term median of a triangle.
- Ask students: How many medians can a triangle have? *(three)*
- Instruct them to draw a ΔABC and median AD as given in the course book. Ask them to draw and mark the other two medians as well.

Altitude of a triangle

- Explain the term altitude of a triangle with the help of the course book. Explain the term base with respect to altitude.
- Explain the difference between the altitude and the median.
- Ask students: How many altitudes can a triangle have? (*three*)
- Instruct them to draw a ΔABC and altitude AD to the base BC. Ask them to draw and mark the other two altitudes as well.
- Check if they get a diagram like this.



- Check if they have produced side AC to draw altitude BE and side AB to draw altitude CF.
- Ask students to attempt the Quick Check exercise.
- Instruct them to study the solved examples given in the Guided Learning section in the course book for types of triangles, sum of angles of a triangle, exterior angles of a triangle and length of sides of a triangle.
- Ask them to attempt the Tryout for these topics in the course book.

2. Extension activity for altitudes and medians of triangles

- Divide the class into convenient groups.
- Instruct each group to draw one big triangle of each type—equilateral triangle, isosceles triangle, scalene triangle, acute-angled triangle, obtuse-angled triangle and right-angled triangle—on separate sheets of paper.
- Ask them to draw the medians and the altitudes of these triangles. Instruct them to use different coloured pencils for medians and altitudes

- At the end of the task, ask them these questions:
  - Which triangle has one of its sides as the altitude? *(right-angled triangle)*
  - In which triangles did the medians and altitudes occur on the same line/coincide? *(equilateral triangle)*

Pythagoras' theorem

- Instruct students to attempt the Activity to derive the Pythagoras' theorem.
- State the Pythagoras' theorem as given in the course book.
- Ask students to complete the Activity to verify the Pythagoras' theorem.

Corollary 1

- State and prove corollary 1 of the Pythagoras' theorem as given in the course book.

Corollary 2

- State and prove corollary 2 of the Pythagoras' theorem as given in the course book.

Converse of Pythagoras' theorem

- State and explain the converse of the Pythagoras' theorem as given in the course book.
- Ask students to construct triangles of the specified measures given in the course book to verify the converse of Pythagoras' theorem.

Pythagorean triples

- Explain the concept of Pythagorean triples using the examples given in the course book.
- Explain the conditions for numbers to form a Pythagorean triple. (*They should be integers. One integer should be bigger than the other.*)
- Ask students to write down different pairs of integers and form Pythagorean triples with them.
- Instruct them to study the solved examples in the Guided Learning section based on the Pythagoras' theorem.
- Ask them to complete the Tryout given in the course book for the Pythagoras' theorem and its practical applications.

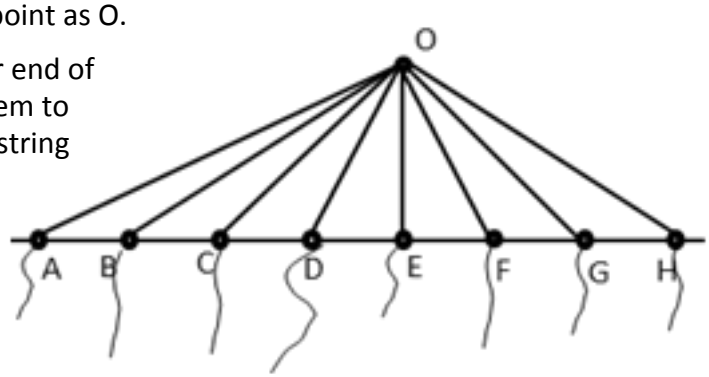
More suggestions for extension activities

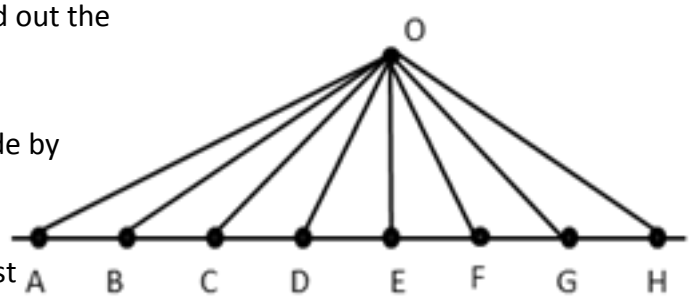
3. Activity (To verify corollary 2 of the Pythagoras' theorem)

You need a spool of thick string/wool and a pair of scissors for the activity. Cut out long pieces of different coloured string/ wool and keep them ready.

- Draw a line on the floor of the classroom.
- Mark different points on this line and name them A, B, C and so on. Instruct the required number of students to occupy these positions on the line.

- Draw a straight (vertical) line from any of the points on the line to a point outside it. Take all the pieces of string with you and stand at the other end of this vertical line. Hold one end of each string. Let their other ends hang down.
- Instruct the class to note that you and only one student are standing on points that fall on the same vertical line. (As per the reference figure given here, you and the student at point E would be on the same vertical line.)
- Ask a student volunteer to mark your point as O.
- Instruct the volunteer to pick the other end of the strings you are holding and give them to the students standing on the line, one string per student.
- Ensure that you and these students hold the strings taut.
- Instruct the volunteer to cut off the extra portions from all the strings.
- Now, leave your end of the strings. Ask the students holding the other ends to measure their respective strings and write the lengths on the board as OA = ____ cm, OB = _____ cm, and so on.
- Let the class compare the lengths to find out the string of the shortest length. (As per the diagram, it is OE.)
- Ask a student to measure the angle made by the line OE on the line AH. (*90*°).
- Help students to conclude that OE is perpendicular to AH and it is the shortest line segment from a point outside it.





Question Bank

1. Choose the correct option.

a. A right-angled triangle is also isosceles. Each of its equal angles is

i. 90° ii. 45° iii. 60° iv. 30°

b. A rectangle is divided along a diagonal. The two triangles formed are

i. equilateral and acute-angled

ii. isosceles and acute-angled

iii. scalene and right-angled

iv. isosceles and right-angled

c. A square is divided along a diagonal. The two triangles formed are

i. equilateral and acute-angled

ii. isosceles and acute-angled

iii. scalene and right-angled

iv. isosceles and right-angled

d. The angles of a triangle are in the ratio 1:1:4. The angles are

i. 10°, 10°, 40° ii. 20°, 20°, 80°

iii. 30°, 30°, 120° iv. 30°, 60°, 90°

e. An equilateral triangle is cut along the bisector of a vertex. The two triangles formed will have the angles of

i. 30°, 60°, 90° ii. 60°, 60°, 60°

iii. 45°, 45°, 90° iv. 30°, 30°, 120°

2. Fill in the blanks.

a. The prefix tri- means _____ .

b. A triangle in which no two sides are equal is called _____ .

c. In an equilateral triangle, all angles measure _____ .

d. The sum of the three angles of a triangle is _____.

e. In an isosceles triangle, the angles opposite the equal sides are _____ .

f. In a right-angled triangle, the hypotenuse is the _____ (longest/shortest) side.

g. The shortest distance from a point to a line is the _____ distance between them.

h. Complete the Pythagorean triple: 5, _____ , 13.

3. Answer the following.

a. Say whether triangles are possible with the following sides:

i. 4 cm, 5 cm, 6 cm

ii. 2 cm, 5 cm, 8 cm

iii. 25 cm, 15 cm, 35 cm

iv. 12 cm, 18 cm, 20 cm

b. Say whether triangles are possible with the following angles:

i. 30°, 50°, 90° ii. 35°, 35°, 110°

iii. 10°, 10°, 160° iv. 65°, 70°, 75°

c. Classify triangles with the following sides as equilateral, isosceles and scalene:

i. 4 cm, 7 cm, 10 cm

ii. 7.2 cm, 7.2 cm, 7.2 cm

iii. 5 cm, 9 cm, 9 cm

iv. 3.5 cm, 4.5 cm, 6.5 cm

d. Classify triangles with the following angles as acute-, obtuse- and right-angled:

i. 60°, 60°, 60° ii. 30°, 60°, 90°

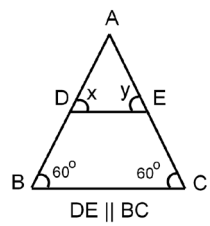
iii. 30°, 40°, 110° iv. 45°, 55°, 80°

e. The ratio of the sides of a triangle is 2:3:4. If the shortest side is 2.5 cm long, find the perimeter of the triangle.

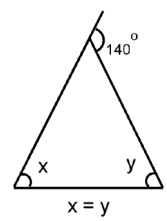
f. In an isosceles triangle, one angle is equal to 100°. Find the other angles.

g. Find the unknown angles in the given figures.

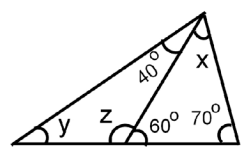
i.



ii.



iii.



h. Prove that in a right-angled triangle, the hypotenuse is the longest side.

i. A right-angled triangle has a hypotenuse of 17 cm and a side of 15 cm. Find the third side.

j. A man travelled 24 km in a straight line from south to north and then 7 km from there to east also in a straight line. How far will he be from the starting point?

k. A rectangular garden of length 8 m and breadth 6 m has a narrow path running along its diagonal. What is the length of the pathway?

l. A vertical pillar of length 40 m on a bridge is supported by a slanting iron rod. One end of the rod is fastened to the tip of the pillar and the other end to the bridge at a distance of 30 m from the pillar. What is the length of the iron rod? Draw a diagram to represent this.

Answer Key to the Question Bank

1. a. ii b. iii c. iv d. iii e. i **2.** a. 3 b. scalene c. 60° d. 180° e. equal f. longest g. perpendicular h. 12 **3.** a. i. possible ii. not possible iii. possible iv. possible b. i. not possible ii. possible iii. possible iv. not possible c. i. scalene ii. equilateral iii. isosceles iv. scalene d. i. acute-angled ii. right-angled iii. obtuse-angled iv. acute-angled e. 11.25 cm f. 40°, 40° g. i. 60°, 60° ii. 70°, 70° iii. 50°, 20°, 120° h. Corollary 1 of the Pythagoras' theorem given in the course book i. 8 cm j. 25 km k. 10 m l. 50 m

Answer Key—Properties of Triangles Worksheet

1. a. right-angled b. obtuse c. acute d. acute 2. a. scalene b. isosceles c. equilateral d. scalene 3. a. scalene b. equilateral c. scalene d. scalene 4. a. 80° b. 90° c. 120° d. 60° 5. 30°, 60°, 90°; right-angled 6. $\angle x = 80°$, $\angle y = 70°$, $\angle z = 110°$ 7. a. yes b. no 8. a. 5 cm b. 12 cm c. 1.2 cm 9. 10 cm 10. a. no b. 45°

PROPERTIES OF TRIANGLES — Worksheet

1. The three angles of a triangle are given. Classify the triangle as acute, obtuse or right-angled:
   a. 35°, 55°, 90°: _____ b. 30°, 110°, 40°: _____
   c. 53°, 85°, 42°: _____ d. 60°, 60°, 60°: _____

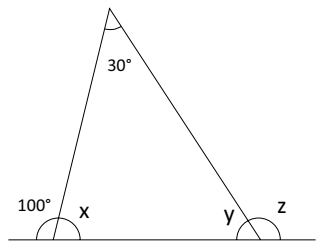
2. Measures of three sides of a triangle are given. Classify the triangle as equilateral, isosceles or scalene:
   a. 35 cm, 45 cm, 55 cm: _____ b. 7 cm, 4 cm, 7 cm: _____
   c. 12 cm, 12 cm, 12 cm: _____ d. 4 cm, 6 cm, 9 cm: _____

3. The three angles of a triangle are given. Classify the triangle as equilateral, isosceles or scalene:
   a. 50°, 20°, 110°: _____ b. 60°, 60°, 60°: _____
   c. 40°, 90°, 50°: _____ d. 25°, 130°, 25°: _____

4. Two angles of a triangle are given. Find the third angle:
   a. 35°, 65°: _____ b. 45°, 45°: _____
   c. 18°, 42°: _____ d. 30°, 90°: _____

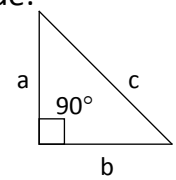
5. In a triangle, the angles are in the ratio 1 : 2 : 3. Find the angles. What type of a triangle is it? _____
   _____

6. Find $\angle x$, $\angle y$ and $\angle z$ in the following figure:



7. Can a triangle be constructed with sides of the following lengths?
   a. 10 cm, 8 cm, 15 cm: _____ b. 22 cm, 33 cm, 56 cm: _____

8. Two sides of a right-angled triangle are given. Find the third side:
   a. $a = 3$ cm; $b = 4$ cm; $c = ?$ _____
   b. $b = 35$ cm; $c = 37$ cm; $a = ?$ _____
   c. $a = 0.5$ cm; $c = 1.3$ cm; $b = ?$ _____



9. Find the length of the diagonal of a rectangle whose length and breadth are 8 cm and 6 cm respectively.
   _____

10. a. Can an obtuse angled triangle be right-angled? _____
    b. If a right-angled triangle is isosceles, find its equal angles: _____